

# Relay Selection with Network Coding in Two-Way Relay Channels

Yonghui Li, *Member, IEEE*, Raymond H. Y. Louie, *Student Member, IEEE*, and Branka Vucetic, *Fellow, IEEE*

**Abstract**—In this paper, we consider the design of joint network coding (NC) and relay selection (RS) in two-way relay channels. In the proposed schemes, two users first sequentially broadcast their respective information to all the relays. We propose two RS schemes, a single relay selection with NC and a dual relay selection with NC. For both schemes, the selected relay(s) perform NC on the received signals sent from the two users and forward them to both users. The proposed schemes are analyzed and the exact bit error rate (BER) expressions are derived and verified through Monte Carlo simulations. It is shown that the dual relay selection with NC outperforms other considered relay selection schemes in two-way relay channels. The results also reveal that the proposed NC relay selection schemes provide a selection gain compared to a NC scheme with no relay selection, and a network coding gain relative to a conventional relay selection scheme with no NC.

**Index Terms**—Bidirectional Relay, cooperative communications, decode and forward, network coding, relay networks, selective relaying, two-way relay channels

## I. INTRODUCTION

Wireless channels typically suffer from time varying fading caused by multipath propagation and Doppler shifts, resulting in serious performance degradation. Diversity has been an effective technique in combating channel fading. Recently, a new form of diversity technique, called user cooperative diversity [1], has been proposed for wireless networks. The idea is to allow users to communicate cooperatively by sharing their antennas to achieve a spatial diversity gain. The use of relay aided transmission is one example of a practical cooperative diversity technique. In a relay system, the source sends its information to the relays. The relays then process the received signals, and forward them to the destination. At the destination, by properly combining the received signals sent from the source and relays, cooperative diversity can be achieved. It has been shown that cooperative communications can dramatically improve the system capacity and performance [3], [4].

To further improve the network capacity, the application of network coding (NC) [2] in wireless relay networks has recently drawn significant attention. In particular, NC has

been studied in multiple access, multicast and two-way relay channels, where two users communicate with each other with the help of relays [5]–[9]. Some physical layer NC schemes, joint network-channel coding and scheduling algorithms, etc., have been proposed [5]–[9], [15]–[20]. It has been shown that properly designed NC can achieve significant capacity improvement in cooperative wireless networks.

Most of the current work on two-way relay channels considers the use of a single relay node to aid communication in the system [5]–[9]. In this paper, we consider a two-way relay system with multiple relay nodes. In multiple relay networks, if all relays participate in the relayed transmission, it is usually assumed that they transmit on orthogonal channels so that they do not cause interference to each other [3], [4]. Relaxing the orthogonality constraint can lead to a capacity increase with an increased system complexity. To overcome these problems, relay selection algorithms using various relay protocols, such as amplify and forward, decode and forward (DAF), and their variations, have been proposed to facilitate system design for one way non-orthogonal multiple relay networks [10]–[12], [27]. A commonly used relay selection strategy in one way relay networks is to select a single best relay, which has the optimal end to end performance or capacity among all relays [10]–[12], or among all relays whose received signal-to-noise ratios (SNRs) are larger than a threshold [27]. It was shown that the single relay selection can achieve the full spatial diversity order as if all relays are used. Furthermore, the system bit error rate (BER) performance and capacity compared to all-participation relaying schemes is improved [3], [4].

In this paper, we consider the design of relay selection for two-way relay channels. In [21], an interesting relay selection scheme was proposed for two-way relay channels. The relay selection criterion was to maximize the weighted sum rate for any bidirectional rate pair on the boundary of the achievable rate region. It was shown that the probability that there exists one relay node which achieves the optimal rate pair decreases with increasing the number of relay nodes. The optimal relay selection criterion decides for any rate pair individually and the optimal rate region can be achieved by time-sharing of different relay nodes.

In this paper, we propose practical relay selection schemes for two-way relay channels, designed to minimize the average sum bit error rate (BER) of the two end users in two-way relay channels. We consider a decode and forward relaying protocol for information forwarding. To improve the spectral efficiency and error performance of bidirectional relayed transmission, we combine relay selection (RS) and network

Copyright (c) 2010 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org.

This work was supported in part by the Australia Research Council under Grant DP0985140, DP0877090 and LP0991663.

Manuscript received March 2 2010, revised June 20 2010 and accepted July 28 2010. The review of this paper was coordinated by Dr. Chau Yuen.

The authors are with the Centre of Excellence in Telecommunications, School of Electrical & Information Engineering, University of Sydney, Sydney, NSW 2006, Australia. Email: {lyh, rlouie, branka}@ee.usyd.edu.au, Tel:+61 2 9351 2236, Fax: 61 2 9351 3847.

coding, and develop efficient joint relay selection and network coding schemes (RS-NC). Specifically, we propose two RS-NC schemes, referred to as the single relay selection with NC (S-RS-NC) and dual relay selection with NC (D-RS-NC) schemes. In the proposed schemes, both users first sequentially send their respective information to all the relays. Based on certain selection criteria, a single relay or two relays are selected for transmission. The selected relay(s) decode each user's signals, perform NC and then broadcast the signals to both users. To facilitate the analysis and selection process, we propose some simple selection criteria for both relay selection schemes. Specifically, for the single relay selection, inspired by the fact that the overall error probability of two users is dominated by the worst user, we propose a near-optimum *Min-Max* selection criterion for the S-RS-NC scheme, such that the BER of the worst user among the two users is optimized. That is, a single relay, which minimizes the instantaneous BER of the worst of the two users, will be selected in the proposed S-RS-NC scheme. Simulation results confirm that the *Min-Max* selection criterion achieves almost exactly the same performance as the optimal single relay selection criterion based on the minimization of the average sum BER of two users. During the revision of this paper, we have been informed that a similar *Min-Max* selection criterion for two-way multiple antenna relay network has been proposed in [28], [29], but no closed-form BER expressions were derived. In this paper, we will derive the exact closed-form and asymptotic BER expressions, and the analytical results are verified by Monte-Carlo simulations.

On the other hand, as revealed by the information-theoretic results in [21] that the bidirectional communication is characterized by a two-dimensional rate pair, the optimal relaying may be achieved by time-sharing of multiple relays. Motivated by this result, in this paper we propose another dual-relay selection scheme, referred to as the D-RS-NC. For the optimal D-RS-NC scheme, the destination needs to do an exhaustive search to find the optimal pair of two relays. This procedure is very complex, and requires coordination between the two end users, and also involves significant amounts of feedback. To facilitate the selection process, we propose a simple selection criterion, named *Double-Max* criterion. In this criterion, we select one best relay for each user. Here, the best relay for the user  $k$ ,  $k=1, 2$ , means the relay, which has the best link quality to the user  $k$ . The best relays for two users could be the same or different. If the best relays for two users are the same, then only a single relay is selected for transmission. Otherwise, two different relays will be used. The proposed selection scheme is very simple and can be easily implemented.

The performance of the proposed RS-NC schemes is analyzed and verified by Monte Carlo simulations. Results show that both RS-NC schemes can achieve the full diversity order as if all relays are used. Results also show that the dual relay selection is superior to the single relay selection and outperform other considered relay selection schemes in two-way relay networks. This is different from the conventional one way relay networks [10]–[12] where the single relay selection is the optimal selection strategy. We also compare the performance of RS-NC schemes with other conventional

schemes. Results show that the combined relay selection and NC provides a selection gain compared to the pure NC scheme with no relay selection, and a network coding gain relative to the conventional relay selection with no NC scheme. This implies that a properly combined network coding and relay selection can improve both system performance and spectral efficiency.

The rest of the paper is organized as follows. The system model is described in Section II. In Section III, we propose several relay selection schemes for two-way relay channels. The performances of these schemes are analyzed and compared with the conventional schemes. The results are verified by simulations in Section IV. In Section V, we draw the conclusions.

## II. SYSTEM MODEL

In this paper, we consider a general two-hop two-way relay network, where user 1 and user 2 exchange their information with the help of  $N$  relays. For simplicity, in this paper, we consider the BPSK modulation. The extension to other modulation schemes is straightforward.

Let  $b_1(k)$  and  $b_2(k)$  represent the  $k$ -th information bit transmitted by user 1 and 2, respectively, and  $s_1(k)$  and  $s_2(k)$  denote the corresponding modulated symbols. The overall transmission can be divided into three steps. In the first two time slots, user 1 and user 2 send their respective information symbol  $s_1(k)$  and  $s_2(k)$  to all relays. The corresponding received signal at the  $i$ -th relay transmitted from user  $j$ , denoted by  $y_{u_j, r_i}(k)$ ,  $j=1, 2$ ,  $i=1, 2, \dots, N$ , can be expressed as

$$y_{u_j, r_i}(k) = \sqrt{\bar{p}_{u_j}} h_{u_j, r_i}(k) s_j(k) + n_{u_j, r_i}(k) \quad (1)$$

where  $\bar{p}_{u_j}$ ,  $j=1, 2$ , is the average transmission power at user  $j$  and  $h_{u_j, r_i}(k)$  is the fading coefficient between user  $j$  and relay  $i$ . In this paper, we assume that all fading coefficients are modeled as zero-mean, unit variance, independent circular symmetric complex Gaussian random variables. Furthermore,  $n_{u_j, r_i}(k)$  is a zero mean complex Gaussian random variable with a noise variance of  $\sigma_n^2$ . In this paper, we assume that all noise processes have the same variance without loss of generality.

In the final step, the relays process the received signals and transmit it to the two users. Specifically, the relays decode the received signals from users 1 and 2, and perform network coding by XORing the two decoded bits  $b_1(k)$  and  $b_2(k)$ . Let  $b_r(k)$  represent the binary summation of  $b_1(k)$  and  $b_2(k)$ , and  $s_r(k)$  denote the corresponding modulated symbol. Let  $x_{r, i}(k)$  be the  $k$ -th signal transmitted from the  $i$ -th relay, expressed as

$$x_{r, i}(k) = \sqrt{\bar{p}_{r, i}} s_r(k) \quad (2)$$

where  $\bar{p}_{r, i}$  is the average transmission power at relay  $i$ . Relay  $i$  then broadcasts this signal to both users 1 and 2. The corresponding received signal at the  $j$ -th user receiver can be written as

$$y_{r, i, u_j}(k) = h_{r, i, u_j}(k) x_{r, i}(k) + n_{r, i, u_j}(k) \quad (3)$$

where  $h_{r, i, u_j}(k)$  is the fading coefficient between relay  $i$  and user  $j$ .

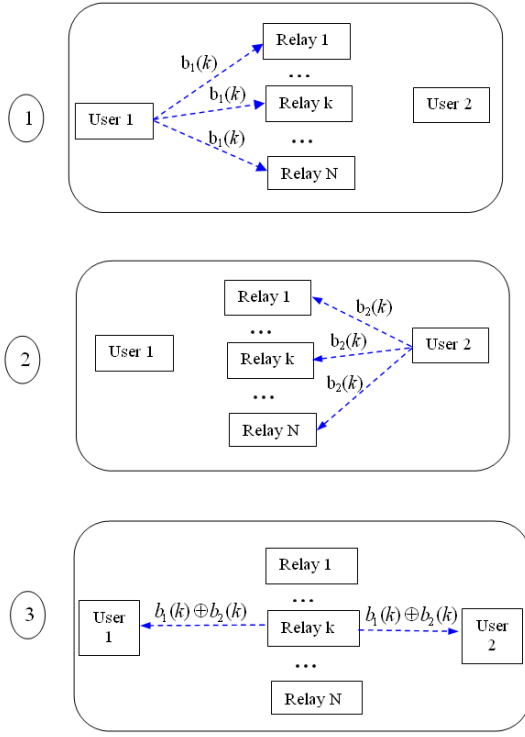


Fig. 1. Three step transmission of single relay selection with network coding (S-RS-NC) in two-way relay networks

After receiving signals from the relays, each user then decodes the received signals and estimates  $b_r(k) = b_1(k) \oplus b_2(k)$ . Let  $\hat{b}_{r,1}(k)$  and  $\hat{b}_{r,2}(k)$  represent the estimated  $b_r(k)$  at user 1's and user 2's receivers, respectively. Since user  $i$  already knows its own transmitted bit  $b_i(k)$ , it can recover other user's bit by simply XORing  $\hat{b}_{r,i}(k)$  with its own bit  $b_i(k)$ . For example, user 1 can obtain an estimation of  $b_2(k)$ , denoted by  $\hat{b}_2(k)$ , by performing

$$\hat{b}_2(k) = b_{r,1}(k) \oplus b_1(k). \quad (4)$$

### III. RELAY SELECTION WITH NETWORK CODING IN TWO-WAY RELAY CHANNELS

In this section, we propose several joint relay selection and network coding (RS-NC) schemes for two-way relay channels. We consider a decode and forward relaying protocol, where the relays decode the received signals before forwarding them to the destination. Similar to [24]–[26], in this paper, we focus on the scenario where the link from the source to relay is much more reliable than the link from the relay to destination and as such the overall error probability from the source to the destination is dominated by the error probability from the relay to the destination and the contribution of decoding errors at the relay to the overall performance can be ignored. For simplicity of analysis, we assume error-free decoding at the relays. This is representative of many practical scenarios, such as when the source transmission power is large compared to the relay transmission power. One example includes two base stations that exchange their information through a relay station.

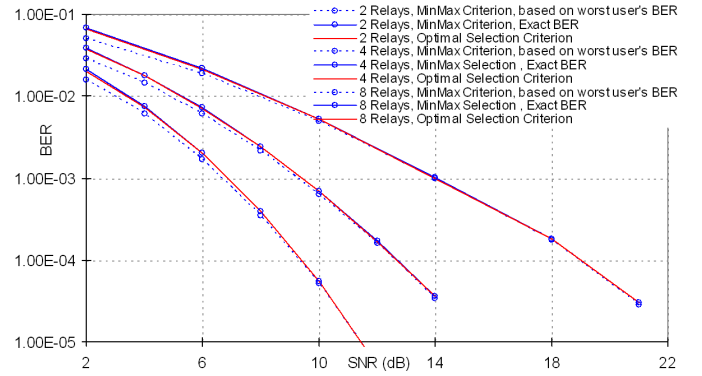


Fig. 2. The comparison of BER based on worst user's BER and exact BER using MinMax selection criterion and the exact BER based on the optimal selection criterion for single relay selection with NC

To facilitate the BER proofs in this section, note that after integration by parts, we have

$$E_x \left[ Q \left( \sqrt{bX} \right) \right] = \frac{\sqrt{b}}{2\sqrt{2\pi}} \int_0^\infty \frac{e^{-bx/2} F_X(x)}{\sqrt{x}} dx \quad (5)$$

where  $Q(\bullet)$  is the  $Q$  function and  $F_X(x)$  is the cumulative distribution function (CDF) of  $X$ . In addition, if the first order expansion of the probability density function (PDF) of  $X$  can be written in the form

$$f_X(x) = \frac{ax^N}{\gamma^{N+1}} + o(x^{N+\epsilon}), \epsilon > 0 \quad (6)$$

at high SNR, the asymptotic BER is given by [23]

$$E_x \left[ Q \left( \sqrt{bX} \right) \right] = \frac{2^N a \Gamma(N + 3/2)}{\sqrt{\pi} (N + 1)} (b\gamma)^{-(N+1)} + o(\gamma^{-(N+1)}) \quad (7)$$

#### A. Single Relay Selection with Network Coding (S-RS-NC)

In this section, let us first consider an RS-NC scheme, referred to as the single RS-NC (S-RS-NC) scheme, where only a single relay, which optimizes the system performance, is selected. Its block diagram is shown in Fig. 1. For different selection criteria, the selection decision procedures are different, so we will discuss it separately for each scheme.

Realizing the fact that the average sum BER of two end users is dominated by the worst user, in this paper, we consider a simplified selection criterion for which the instantaneous BER of the worst user among two users is minimized. We refer to such a selection criterion as the *Min-Max* selection criterion. Fig. 2 compares the performance of an approximate BER using only the worst user's BER and exact two users' BER based on the *Min-Max* selection criterion, as well as the exact two users' BER based on the optimum selection criterion for which the average sum BER of two users is minimized. To facilitate the simulation of the optimal selection criterion, we use a Chernoff bound to approximate the  $Q$ -function when determining the optimal relay. In this figure, we assume that the average SNR in the link from relay  $i$  to user  $j$  for  $i = 1, \dots, N$  and  $j = 1, 2$ , are all equal. As can be seen

from the figure, the worst user's BER is almost the same as the exact two users' BER at medium to high SNR regions. We can also observe that the *Min-Max* selection criterion achieves almost exactly the same BER performance as the optimal relay selection at all SNR range. This confirms that the *Min-Max* selection criterion is almost equivalent to the optimal selection criterion for the single relay selection.

Now let us calculate the BER expression of the S-RS-NC scheme using the *Min-Max* selection criterion. Let  $\hat{b}_{r,1}(k)$  and  $\hat{b}_{r,2}(k)$  denote the estimation of  $b_r(k)$  at user 1's and user 2's receivers, respectively. It can be easily verified that any error in  $\hat{b}_{r,j}(k)$ ,  $j = 1, 2$ , will result in a bit error at user  $j$ 's receiver. For example, if  $\hat{b}_{r,1}(k)$  is in error, then the estimation of  $b_2(k)$  at user 1's receiver, given by  $\hat{b}_2(k) = \hat{b}_{r,1}(k) \oplus b_1(k)$ , will also be erroneous, because user 1 knows  $b_1(k)$  perfectly. This means that the error probability of  $b_2(k)$  at user 1's receiver is exactly the same as the error probability of  $\hat{b}_{r,1}(k)$ . If we let  $P_{r_i, u_j}$  be the BER in the link from relay  $i$  to user  $j$ , then the average BER of two users in this link, denoted by  $P_{r_i}$ , is equal to  $P_{r_i} = (P_{r_i, u_1} + P_{r_i, u_2})/2$ . As discussed before, it can also be approximated by using the BER of the worst user as follows

$$P_{r_i} = \frac{1}{2}(P_{r_i, u_1} + P_{r_i, u_2}) = \frac{1}{2}\max\{P_{r_i, u_1}, P_{r_i, u_2}\} \quad (8)$$

Let  $\gamma_{ij} = \bar{p}_{r,i}/\sigma_n^2$  represent the average SNR in the link from relay  $i$  to user  $j$ . We also assume that all  $\gamma_{ij}$  for  $i = 1, \dots, N$ ,  $j = 1, 2$ , are the same and equal to  $\gamma_{rd} = \bar{p}_r/\sigma_n^2$ , where  $\bar{p}_r$  is the total relay transmission power.

Let  $P_{r_i, u_j}(\gamma_{rd}|h_{r_i, u_j}(k))$  represent the instantaneous BER, in the link from relay  $i$  to user  $j$ . It is given by

$$P_{r_i, u_j}(\gamma_{rd}|h_{r_i, u_j}(k)) = Q\left(\sqrt{2\gamma_i^{u_j}(k)}\right) \quad (9)$$

where  $\gamma_i^{u_j}(k) = \gamma_{rd}|h_{r_i, u_j}(k)|^2$  is the instantaneous received SNR in the link from relay  $i$  to user  $j$ .

If we let  $z = \gamma_i^{u_j}(k)$ , the PDF of  $z$  is given by

$$f_Z(z) = \gamma_{rd}^{-1} \exp(-z\gamma_{rd}^{-1}). \quad (10)$$

Let  $P_{r_i, \max}(\gamma_{rd}|h_{r_i, \min}(k)) = \max\left\{P_{r_i, u_1}(\gamma_{rd}|h_{r_i, u_1}(k)), P_{r_i, u_2}(\gamma_{rd}|h_{r_i, u_2}(k))\right\}$  denote the error probability of the worst user for the signals transmitted from relay  $j$ , where  $|h_{r_i, \min}(k)| = \min\{|h_{r_i, u_1}(k)|, |h_{r_i, u_2}(k)|\}$ . Then based on the *Min-Max* selection criterion, among all relays, a single relay, denoted by  $S$ , is selected such that the instantaneous BER of the worst user is minimal. That is,

$$\begin{aligned} S &= \arg \min_i \{P_{r_i, \max}(\gamma_{rd}|h_{r_i, \min}(k))\} \\ &= \arg \min_i \{\max\{P_{r_i, u_1}(\gamma_{rd}|h_{r_i, u_1}(k)), P_{r_i, u_2}(\gamma_{rd}|h_{r_i, u_2}(k))\} \\ &\quad, i = 1, \dots, N\} \end{aligned} \quad (11)$$

and other unselected relays will stay in the idle states.

Since  $Q(x)$  is a monotonic decreasing function of  $x$ , the *Min-Max* criterion in (11) can be further written as

$$\begin{aligned} S &= \arg \max_i \{\gamma_i^{\min}(k)\} \\ &= \arg \max_i \{\min\{\gamma_i^{u_1}(k), \gamma_i^{u_2}(k)\}, i = 1, \dots, N\} \\ &= \arg \max_i \{\min\{|h_{r_i, u_1}(k)|^2, |h_{r_i, u_2}(k)|^2\}, i = 1, \dots, N\} \end{aligned} \quad (12)$$

where  $\gamma_i^{\min}(k) = \min\{\gamma_i^{u_1}(k), \gamma_i^{u_2}(k)\}$  is the minimum of the two instantaneous SNRs of the links from relay  $i$  to the two users. That is

$$\begin{aligned} \gamma_i^{\min}(k) &= \min\{\gamma_i^{u_1}(k), \gamma_i^{u_2}(k)\} \\ &= \gamma_{rd} \min\{|h_{r_i, u_1}(k)|^2, |h_{r_i, u_2}(k)|^2\}. \end{aligned} \quad (13)$$

Let  $z_{\min} = \gamma_i^{\min}(k)$ . From Eqs. (10) and (13), the PDF of  $z_{\min}$  is given by [13]

$$f_{Z_{\min}}(z) = 2\gamma_{rd}^{-1} \exp(-2\gamma_{rd}^{-1}z). \quad (14)$$

Then the exact and asymptotic BER of the S-RS-NC scheme is given in the following theorem.

**Theorem 1** The average sum BER of the S-RS-NC scheme using the *Min-Max* selection criterion is given by

$$P^{S-RS-NC}(\gamma_{rd}) \approx \frac{1}{4} \sum_{p=0}^N \binom{N}{p} (-1)^p \frac{1}{\sqrt{1 + 2p\gamma_{rd}^{-1}}}. \quad (15)$$

It can be further approximated at high SNR as

$$P^{S-RS-NC}(\gamma_{rd}) \approx \frac{2^{N-2}\Gamma(N + \frac{1}{2})}{\sqrt{\pi}} \gamma_{rd}^{-N} + o(\gamma_{rd}^{-N}) \quad (16)$$

*Proof:* See Appendix A.

From the above equation, we can see that a diversity order of  $N$  can always be achieved by the proposed S-RS-NC scheme in a two-way relay network with  $N$  relay nodes.

Next, let us discuss the selection decision procedure for S-RS-NC. Similar to [30], to perform selection at the relay nodes, each relay needs to listen to the request-to-send (RTS) and the clear-to-send (CTS) packets from two source nodes, respectively. Based on that, each relay node estimates its channel power gains from two source nodes. Then, a backoff timer is set to be inversely proportional to the relaying channel quality  $\gamma_i^{\min}(k) = \min\{\gamma_i^{u_1}(k), \gamma_i^{u_2}(k)\}$ . The best relay with the largest  $\gamma_i^{\min}(k)$  and the smallest backoff timer can occupy the channel first. Thus S-RS-NC based on *Min-Max* criterion can be implemented in a decentralized way.

## B. Dual Relay Selection With Network Coding (D-RS-NC)

In the S-RS-NC scheme, only a single relay is selected for transmission. In this section, we consider another relay selection algorithm, referred to as the dual-RS-NC (D-RS-NC). Again, our selection criterion is to minimize the average sum BER of two users. In the D-RS-NC, one or two out of  $N$  relays are selected for forwarding the network coded signals. Let  $S_{(1)}$  and  $S_{(2)}$  denote the selected relays, which could be the same relay or different relays. Furthermore, to simplify the relay selection process and analysis, we consider a simple dual relay selection algorithm, referred to as the *Double-Max*

**Note:** Among all relays, relay  $S(1)$  has the best link quality to user 1 and relay  $S(2)$  relays has the best link quality to user 2. Since  $\gamma_{s(1)}^u(k)$  and  $\gamma_{s(2)}^u(k)$  are independent, when  $\gamma_{s(1)}^u(k)$  is maximal among all  $\gamma_i^u(k)$ ,  $i=1, \dots, N$ ,  $\gamma_{s(2)}^u(k)$  could be any  $q$ -th ( $q=1, \dots, N$ ) maximal in order among all  $\gamma_i^u(k)$ ,  $i=1, \dots, N$ .

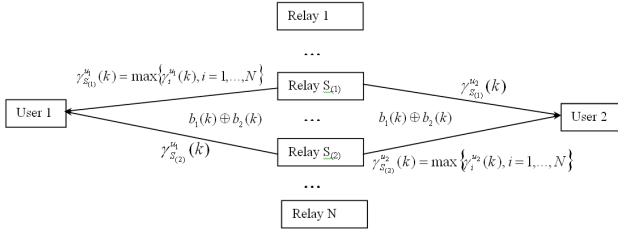


Fig. 3. Block diagram of D-RS-NC based on Double-Max selection criterion

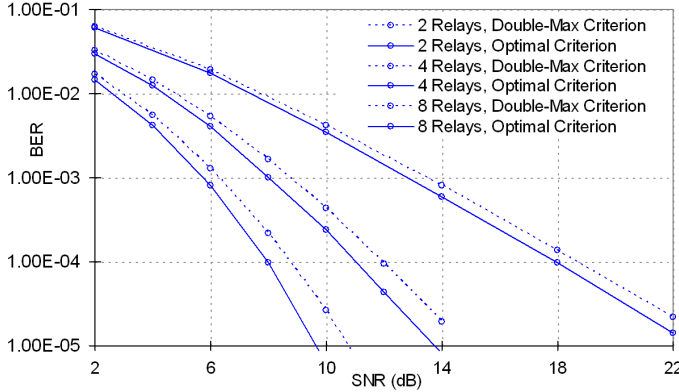


Fig. 4. BER performance comparison of D-RS-NC based on Double-Max and optimal selection criterion

selection criterion, where we select one best relay for each user. Fig. 3 illustrates the relay selection process using the *Double-Max* criterion. Specifically, we select two relays  $S(1)$  and  $S(2)$  such that relay  $S(j)$  out of all relays has the best link quality to the user  $j$ . That is, among all relays, the link from relay  $S(j)$  to user  $j$  has the maximum received SNR at user  $j$ ,  $j=1, 2$ ,

$$S(j) = \arg \max_i \{|h_{r_i, u_j}(k)|^2, i=1, \dots, N\}, j=1, 2 \quad (17)$$

That is,  $|h_{r_{S(j)}, u_j}(k)|$  is the maximum one among all  $|h_{r_i, u_j}(k)|$  for  $i=1, \dots, N$ ,

$$|h_{r_{S(j)}, u_j}(k)|^2 = \max\{|h_{r_i, u_j}(k)|^2, i=1, \dots, N\}, j=1, 2. \quad (18)$$

Fig. 4 compares the D-RS-NC scheme based on the *Double-Max* criterion, with that based on the optimal criterion, which selects optimal relays such that the instantaneous sum BER of two users is minimized. The simulation setup is the same as in Fig. 2. For the optimal criterion, for simplicity we also use a Chernoff bound to approximate the  $Q$ -function when determining the optimal relay(s). It can be seen that there is about 1dB gap between the *Double-Max criterion* and the optimal selection criterion at the BER of  $10^{-4}$ . However, as will be discussed shortly, the *Double-Max* scheme can be implemented in a very simple decentralized way, but the optimal scheme needs to be implemented in a centralized

way. It requires one central node which has channel state information (CSI) of all links to determine the optimal relays minimizing the sum BER and this has to be done by an exhaustive search among all possible pairs of relays. It requires extensive overhead and high implementation complexity. By contrary, the *Double-Max* scheme is much simpler compared to the optimal scheme and is only about 1dB away from the optimal scheme. Also analysis of the optimal selection criterion involves the order statistics of complex  $Q$ -functions which makes it difficult to obtain closed-form expressions. Therefore, in this paper, we only consider the *Double-Max* criterion and derive both exact and asymptotic BER expressions. This will give more insights into the system performance.

Since the two selected relays transmit at the same time, they will interfere with each other. To avoid interference, we consider using an Alamouti space time block code (STBC) technique [14] at the two selected relays  $S(1)$  and  $S(2)$  to facilitate orthogonal transmission. Let  $b_r(k) = b_1(k) \oplus b_2(k)$  and  $s_r(k)$  represent the modulated symbol of  $b_r(k)$ . Let  $s_r^{(j)}(k)$  represent the signals transmitted from relay  $S(j)$  at time  $k$ . Then for STBC, the signal matrix transmitted from  $S(1)$  and  $S(2)$  for the  $k$ -th and  $k+1$ -th symbols, can be written as

$$\begin{bmatrix} s_r^{(1)}(k) & s_r^{(1)}(k+1) \\ s_r^{(2)}(k) & s_r^{(2)}(k+1) \end{bmatrix} = \begin{bmatrix} \sqrt{\bar{p}_{r,1}} s_r(k) & -\sqrt{\bar{p}_{r,2}} s_r^*(k+1) \\ \sqrt{\bar{p}_{r,2}} s_r(k+1) & \sqrt{\bar{p}_{r,1}} s_r^*(k) \end{bmatrix} \quad (19)$$

where  $\bar{p}_{r,j}$ ,  $j=1, 2$ , represents the transmission power at relay  $S(j)$ ,  $\bar{p}_{r,1} + \bar{p}_{r,2} = \bar{p}_r$  and  $\bar{p}_r$  is the total transmission power at the relays. In this paper, we consider equal power allocation between two selected relays. That is,  $\bar{p}_{r,1} = \bar{p}_{r,2} = \bar{p}_r/2$ .

We assume that the channel does not change during the two symbol transmission duration. Following the same receiver process as in the conventional Alamouti STBC system [14], the corresponding received SNR at user  $j$ 's receiver, denoted by  $\gamma^{u_j}(k)$ , can be calculated as

$$\gamma^{u_j}(k) = \frac{1}{2} \gamma_{rd} (|h_{r_{S(1)}, u_j}(k)|^2 + |h_{r_{S(2)}, u_j}(k)|^2) \quad (20)$$

where  $h_{r_{S(i)}, u_j}(k)$  is the fading coefficient from the selected relay  $S(i)$  to user  $j$ . Then we have the following theorem for the BER expression of D-RS-NC scheme.

**Theorem 2** The average sum BER for the D-RS-NC using the *Double-Max* criterion is given by

$$\begin{aligned} P^{D-RS-NC}(\gamma_{rd}) &= \frac{1}{2} E \left( \sum_{j=1}^2 Q \left( \sqrt{2\gamma^{u_j}(k)} \right) \right) \\ &= \frac{\sqrt{\gamma_{rd}}}{2N} \sum_{p=0}^N \binom{N}{p} (-1)^p \frac{1}{\sqrt{\gamma_{rd} + p}} \\ &\quad + (N-1)! (-1)^N \sum_{q=1}^{N-1} \sum_{k=1}^q \sum_{p=q+1}^N \frac{\Psi_0}{\Sigma_0} \end{aligned}$$

and it can be further approximated at high SNR as

$$P^{D-RS-NC}(\gamma_{rd}) \approx \frac{(2^N - 1) \Gamma(N + \frac{1}{2})}{2N \sqrt{\pi}} \gamma_{rd}^{-N} + o(\gamma_{rd}^{-N}) \quad (21)$$

where

$$\Sigma_0 = \prod_{j=1, j \neq k}^q (j-k) \prod_{m=q+1, m \neq p}^N (m-p)(N-P+1)(N-k+1),$$

$$\Psi_0 = \begin{cases} \frac{1}{2} \left( 1 + \sqrt{\frac{c_1}{1+c_1} \frac{c_1}{c_2-c_1}} - \sqrt{\frac{c_2}{1+c_2} \frac{c_2}{c_2-c_1}} \right) & c_1 \neq c_2 \\ \frac{1}{2} \left( 1 - \sqrt{\frac{c_1}{1+c_1}} \left( 1 + \frac{1}{2(1+c_1)} \right) \right) & c_1 = c_2 \end{cases}, \quad (22)$$

$$c_1 = \frac{\gamma_{rd}}{N-k+1}, c_2 = \frac{\gamma_{rd}}{2(N-p+1)}.$$

*Proof:* See Appendix B.

By comparing the asymptotic BER of D-RS-NC in Eq. (21) with the BER of S-RS-NC in Eq. (16), we have

$$P^{D-RS-NC}(\gamma_{rd}) = G_{D/S} P^{S-RS-NC}(\gamma_{rd}) \quad (23)$$

where  $G_{D/S} = \frac{2-2^{-(N-1)}}{N} < 1$  for  $N > 1$  represents the BER reduction of D-RS-NC relative to S-RS-NC.

Next, let us discuss the relay selection process for D-RS-NC. In the D-RS-NC, each user only needs to select a best relay for itself, so the decentralized selection method in the S-RS-NC scheme can also be applied here, but the selection needs to be done in two steps. In each step, one relay is selected for one user. Similar to S-RS-NC, to select a best relay to user 1, each relay sets a backoff timer which is inversely proportional to the relaying channel quality  $\gamma_i^{u_1}(k)$ . The best relay with the largest  $\gamma_i^{u_1}(k)$  and the smallest backoff timer can occupy the channel first and knows that it is the optimal relay to user 1. Similarly, to select a best relay to user 2, each relay sets a backoff timer which is inversely proportional to  $\gamma_i^{u_2}(k)$  and the optimal relay for user 2 can then be selected. Thus D-RS-NC can also be implemented in a decentralized way.

It can be observed from Eq. (23) that unlike the conventional relay networks, where a single relay selection is optimum, in the two-way relay networks, the performance of dual-relay selection with NC is always superior to the single relay selection with NC.

### C. Network Coding without Relay Selection (NC-No-RS)

As a comparison, let us now consider a conventional network coding scheme with no relay selection (NC-No-RS). We assume that all relays transmit on orthogonal channels, so that the receiver can separate them without any interference from each other. This can be done by orthogonal space time block coding or other orthogonal transmission techniques. We should note that the NC-No-RS achieves the same spectral efficiency as the NC with RS schemes. The STBC orthogonal transmission in NC-No-RS is only used to facilitate the receiver processing and it does not increase the frequency bandwidth and reduce the spectral efficiency, compared to the NC with RS schemes.

For a fair comparison, we assume that the total relay transmission power in the NC-No-RS scheme is the same as that in the RS-NC schemes. In this paper, we assume that the power is equally distributed among all relays.

After receiving  $N$  independent signals from  $N$  relays, each user's receiver will then combine all the received signals. The overall received SNR at the  $j$ -th user's receiver at time  $k$ , denoted by  $\gamma_{sum,j}(k)$ ,  $j = 1, 2$ , can be calculated as

$$\gamma_{sum,j}(k) = \frac{\gamma_{rd}}{N} \sum_{i=1}^N |h_{r_i, u_j}(k)|^2 \quad (24)$$

where  $\gamma_{rd} = \bar{p}_r/N_0$ .

**Theorem 3** The average sum BER of NC-No-RS is given by

$$P^{NC-No-RS}(\gamma_{rd}) = \frac{1}{2} - \frac{1}{2\sqrt{\pi}} \sum_{p=0}^{N-1} \frac{(N\gamma_{rd}^{-1})^p \Gamma(p+1/2)}{p!(1+N\gamma_{rd}^{-1})^{(p+1/2)}}. \quad (25)$$

Its asymptotic BER expression at high SNR can be approximated as

$$P^{NC-No-RS}(\gamma_{rd}) \approx \frac{N^N \Gamma(N+1/2)}{2\sqrt{\pi} \Gamma(N+1)} \gamma_{rd}^{-N} + o(\gamma_{rd}^{-N}) \quad (26)$$

By comparing the asymptotic BER of NC-No-RS in Eq. (26) with the BER expression of S-RS-NC shown in Eq. (16), we have

$$P^{S-RS-NC}(\gamma_{rd}) \approx G_{S-RS} P^{NC-No-RS}(\gamma_{rd}) \quad (27)$$

where  $G_{S-RS} = \frac{N!2^{N-1}}{N^N}$  represents the BER reduction of the S-RS-NC compared to the NC-No-RS. The gain comes from the relay selection. It can be verified that  $G_{S-RS} \leq 1$  for  $N > 0$ , and it decreases with increasing  $N$ . This means that a relay selection scheme is always superior to non-selection schemes with all relay participation. The performance gain using relay selection increases as the number of relay increases.

### D. Conventional Relay Selection With No Network Coding (RS-No-NC)

As a comparison, now let us also consider a conventional relay selection (RS) scheme without using network coding in two-way relay networks. To avoid interference, we also assume that all the relays transmit on orthogonal channels, so that they do not interfere with each other. In the RS-No-NC scheme, we select one best relay for each destination user and each selected relay will be used to forward other user's information to its destination user. Specifically, user 1 and user 2 first broadcast their own information to all the relays, separately. One relay, which has the best link quality to user 2 is selected for forwarding user 1's data to user 2, while the another relay which has the best channel to user 1 is selected for forwarding user 2's data to user 1. From the above description, we can see that the RS-No-NC is somewhat similar to the D-RS-NC scheme based on *Double-Max* criterion. Therefore, relay selection is the same as the D-RS-NC scheme. The only difference is that there is no network coding in RS-No-NC scheme and each selected relay only carries one user's information.

For a fair comparison, we also assume that the total transmission power of two selected relays is equal to  $\bar{p}_r$ . Following the similar analysis as for the S-RS-NC scheme, the average



TABLE I  
GAIN OF VARIOUS RELAY SELECTION ALGORITHMS RELATIVE TO THE NC-No-RS

Relay Selection Scheme	BER reduction relative to the NC-No-RS
S-RS-NC	$\frac{N!2^{N-1}}{N^N}$
D-RS-NC	$\frac{(N-1)!(2^N-1)}{N^N}$
Conventional RS-No-NC	$\frac{N!2^N}{N^N}$

BER of RS-No-NC, denoted by  $P^{RS-No-NC}(\gamma_{rd})$ , can be calculated as

$$\begin{aligned}
 P^{RS-No-NC}(\gamma_{rd}) &= \\
 &= \frac{1}{2} E \left( Q \left( \sqrt{\gamma_{N,1}^{max}(k)} \right) + Q \left( \sqrt{\gamma_{N,2}^{max}(k)} \right) \right) \\
 &= \frac{1}{2\sqrt{2\pi}} \int_0^\infty \frac{e^{-x/2}}{\sqrt{x}} \left( 1 - e^{-\gamma_{rd}^{-1}x} \right)^N dx \\
 &= \frac{1}{2\sqrt{2}} \sum_{p=0}^N \binom{N}{p} (-1)^p \left( \frac{1}{2} + p\gamma_{rd}^{-1} \right)^{-1/2} \quad (28)
 \end{aligned}$$

where  $\gamma_{N,j}^{max}(k) = \max\{\gamma_i^{u_j}(k), i = 1, \dots, N\}$  for  $j = 1, 2$  and  $\gamma_i^{u_j}(k) = \gamma_{rd}|h_{r_i,u_j}(k)|^2$ .

Again using the first order expansion of the PDF of  $\gamma_{N,j}^{max}(k)$  and Eq. (7), its asymptotic BER at high SNR is given by

$$P^{RS-No-NC}(\gamma_{rd}) \approx \frac{2^{N-1}\Gamma(N+1/2)}{\sqrt{\pi}} \gamma_{rd}^{-N} + o(\gamma_{rd}^{-N}) \quad (29)$$

By comparing Eq. (29) with Eq. (16), (16) can be rewritten as

$$P^{S-RS-NC}(\gamma_{rd}) = \frac{1}{2} P^{RS-No-NC}(\gamma_{rd}) \quad (30)$$

From the above equation, we can see that the error rate of the S-RS-NC is always half of the conventional RS with no NC. This gain is due to network coding.

Table 1 summarizes the BER reduction of various relay selection schemes relative to the NC with no relay selections (NC-No-RS) and they are illustrated in Fig. 5. They are calculated based on the asymptotic BER at high SNRs, given in Eqs. (16), (21), (26) and (30). It can be seen from the table and figure that the D-RS-NC scheme has the maximum BER reduction among all relay selection schemes, and thus performs best. The S-RS-NC has the second best performance. The conventional RS with no NC is worse than these two RS schemes, but is superior to the NC with no relay selection as the number of relays is greater than six.

#### IV. SIMULATION RESULTS

In this section, we provide simulation results. All simulations are conducted using a BPSK modulation. We assume that the average SNR in the link from relay  $i$  to user  $j$  for  $i = 1, \dots, N$  and  $j = 1, 2$ , are all equal. We consider the scenario where the link from the source to relay is much more reliable than the link from the relay to destination and as such the decoding errors at the relay can be ignored.

In Fig. 6, we first compare the BER performance of RS-NC when selecting various numbers of relays. The optimal RS-NC scheme is to exhaustively search the optimal relay set among

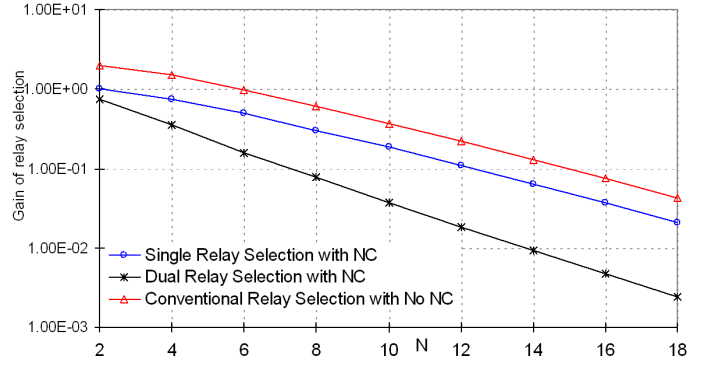


Fig. 5. BER reduction of various relay selection schemes relative to the NC with no relay selection

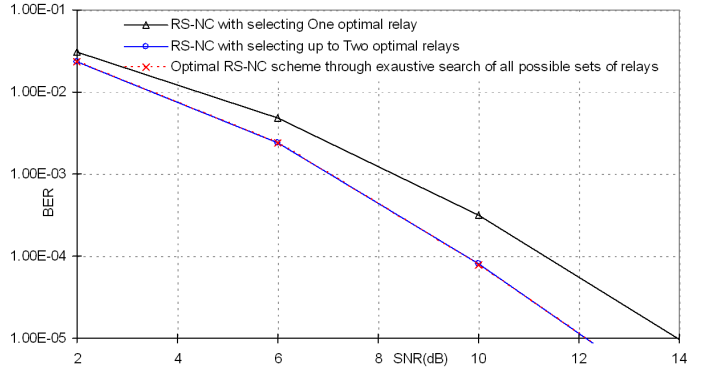


Fig. 6. BER performance comparison with selecting various numbers of relays for 5 relays

all possible sets of relays, where the optimal relay set may contain one, two, three,  $\dots$ , up to  $N$  relays. Obviously, such search has very high complexity, especially when  $N$  is very large. From the figure, we can see that the D-RS-NC, which selects up to two optimal relays performs almost the same as the optimal RS-NC scheme but has much low complexity compared to the optimal relay set selection. This indicates that the D-RS-NC scheme is a near optimal RS-NC scheme in a two-way relay network. It also brings significant gains compared to the S-RS-NC scheme.

Figs. 7-10 compares the BER performance of the single relay selection with NC (S-RS-NC) based on *Min-Max* criterion, dual-relay selection with NC (D-RS-NC) based on *Double-Max* criterion, conventional relay selection with no NC (RS-No-NC) and the NC with no relay selection (RS-No-NC) schemes for various numbers of relays. It can be seen that among all these relay selection schemes, D-RS-NC performs best. This is different from the conventional one way relay networks, where the single relay selection is optimal [11-13]. For the network with two relays, the dual relay selection with NC can provide 0.5dB gain over the S-RS-NC and NC-No-RS, and about 2dBs over the conventional RS-No-NC. The gains are due to the relay selection and they may increase as the number of relays increases. For example, as the number of relays is increased to 16, the gain of the D-RS-NC over the S-RS-NC, RS-No-NC and NC-No-RS are increased to

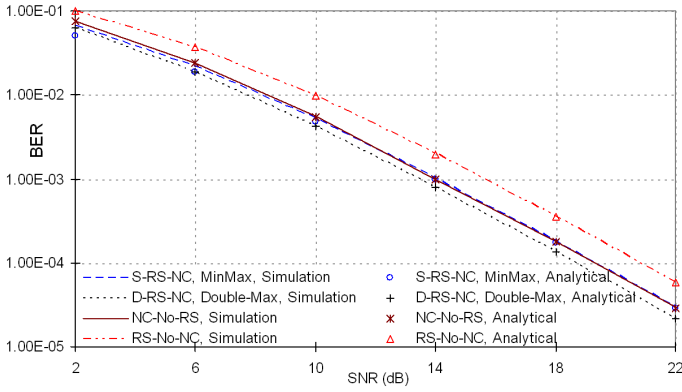


Fig. 7. BER performance comparison for 2 relays

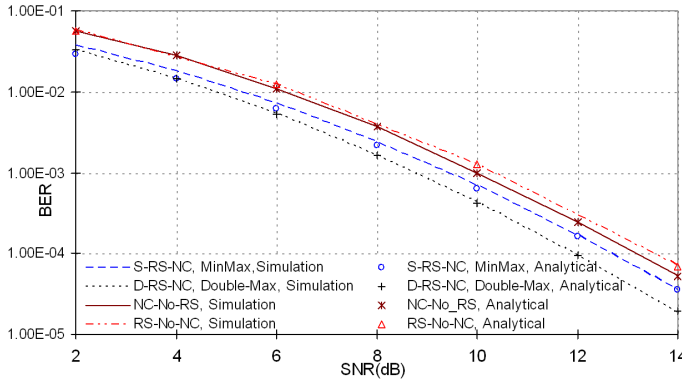


Fig. 8. BER performance comparison for 4 relays

0.5dB, 1dB and 2.5dB, respectively. This is consistent with the analytical results obtained in Section III. However, the gain resulting from relay selection in two-way networks is not as large as relay selection in one way relay networks. This is due to the two-way communication models and use of network coding.

Additionally, we can observe that the single relay selection with NC does not provide any selection gain compared to the NC-No-RS when the number of relays is two. Only when the number of relay is great than two, it provides some gains. This is also different from the relay selection in the conventional one way relay networks, where the relay selection can provide a gain as long as there are more than one relays.

Furthermore, we can also note that the combined relay selection with network coding can provide reasonable gains compared to the conventional relay selection with no network coding under the same complexity. The gains are contributed from the network coding.

Figs. 7-10 also compare the simulation results with the analytical results using the exact BER expressions derived in Section III. It can be seen that for the D-RS-NC, RS-No-NC, NC-No-RS schemes the analytical BER exactly match the simulation results in all SNR ranges. This validates that the BER expressions derived in Section III are accurate. The only BER expression, which has slight deviation from the simulations, is the S-RS-NC scheme. At low SNRs, its analytical BER is slightly better than the simulation results.

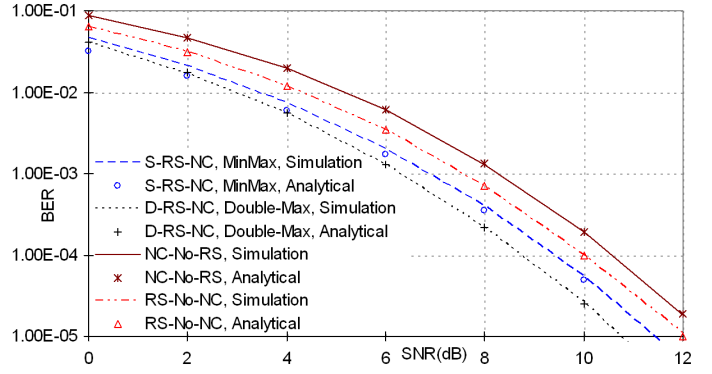


Fig. 9. BER performance comparison for 8 relays

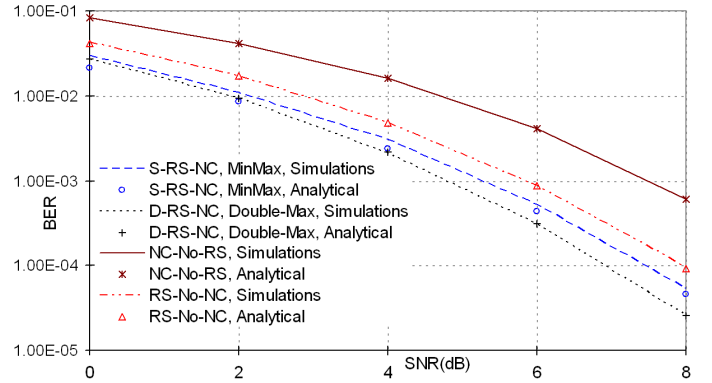


Fig. 10. BER performance comparison for 16 relays

This is because the analytical BER only uses the worst user's BER to approximate the average sum BER of two users. However, as the SNR increases, the analytical BER also matches the simulation results. That is, it is accurate at high SNRs.

From the simulation results, we can see that unlike the conventional relay networks, where a single relay selection is optimum, in two-way relay channels, D-RS-NC outperforms other considered relay selection schemes. However, the D-RS-NC requires strict time synchronization between two selected relays. In contrast, S-RS-NC does not have this requirement. Therefore, there are some implementation complexity and performance tradeoff between these schemes in practical applications.

## V. CONCLUSIONS

In this paper, we combine the network coding and relay selection to improve the transmission efficiency and system performance in two-way relay channels. We proposed several selection schemes that optimize the overall performance of two users, including single relay selection with NC (S-RS-NC) and dual relay selection with NC (D-RS-NC). To simplify the relay selection process, we propose several simplified selection criteria. In particular, for S-RS-NC, we proposed a *Min-Max* selection criterion, so that the BER of the worst user out of two users is optimized, while for D-RS-NC, we developed a simple *Double-Max* criterion, for which we select one best relay for



each user. Unlike the conventional relay selection strategy in one way relay networks, where the optimal selection strategy is to select a single best relay, in two-way relay networks, the dual relay selection outperforms single relay selection and is a near-optimal scheme. It was also shown that a properly combined network coding and relay selection can improve the system performance and efficiency. It provides additional selection gains compared to pure NC schemes with no relay selection, and an extra network coding gain relative to the conventional relay selection with no NC, but the gain due to the relay selection in two-way relay channels is not as larger as for the one way relay channel. Furthermore, in terms of practical implementation, single relay selection is easier to be implemented compared to the D-RS-NC scheme as the later requires synchronization of relay transmission. The performance of D-RS-NC using *Double-Max* criterion is about 1dB away from the optimal criterion. How to design a simple selection criterion which can approach the optimal criterion would be a very interesting problem.

## VI. APPENDIX

### A. Proof of Theorem 1

Let  $\gamma_S^{max-min}(k) = \max\{\gamma_i^{min}(k), i = 1, \dots, N\} = \max\{\min\{\gamma_i^{u1}(k), \gamma_i^{u2}(k)\}, i = 1, \dots, N\}$  represent the minimum value among the instantaneous SNRs from the selected relay  $S$  to users 1 and 2. Based on Eqs. (12) and (14), the PDF of  $\gamma_S^{max-min}(k)$  can be calculated [13]

$$\begin{aligned} f_{max}(x) &= N f_{Z_{min}}(x) (F_{Z_{min}}(x))^{N-1} \\ &= 2N \gamma_{rd}^{-1} \exp(-2\gamma_{rd}^{-1}x) (1 - \exp(-2\gamma_{rd}^{-1}x))^{N-1} \end{aligned} \quad (31)$$

where  $F_{Z_{min}}(x)$  is the CDF of  $\gamma_i^{min}(k)$ , given by

$$F_{Z_{min}}(x) = 1 - \exp(-2\gamma_{rd}^{-1}x).$$

The instantaneous overall BER from the selected relay  $S$  to the two users, denoted by  $P_{rs}(\gamma_{rd}|\gamma_S^{max-min})$ , can be approximated by

$$\begin{aligned} &P_{rs}(\gamma_{rd}|\gamma_S^{max-min}) \\ &= \frac{1}{2} \{P_{rs,u1}(\gamma_{rd}|h_{rs,u1}(k)) + P_{rs,u2}(\gamma_{rd}|h_{rs,u2}(k))\} \\ &\approx \frac{1}{2} \max\{P_{rs,u1}(\gamma_{rd}|h_{rs,u1}(k)), P_{rs,u2}(\gamma_{rd}|h_{rs,u2}(k))\} \\ &= \frac{1}{2} Q(\sqrt{2\gamma_S^{max-min}}). \end{aligned}$$

The average sum BER, denoted by  $P^{S-RS-NC}(\gamma_{rd})$ , can be derived by averaging the above equation with respect to  $\gamma_S^{max-min}$  and is given by

$$\begin{aligned} P^{S-RS-NC}(\gamma_{rd}) &\approx \frac{1}{2} E \left( Q \left( \sqrt{2\gamma_S^{max-min}} \right) \right) \\ &= \frac{1}{4\sqrt{\pi}} \int_0^\infty \frac{e^{-x}}{\sqrt{x}} F_{max}(x) dx \\ &= \frac{1}{4} \sum_{p=0}^N \binom{N}{p} (-1)^p \frac{1}{\sqrt{1+2p\gamma_{rd}^{-1}}} \end{aligned} \quad (32)$$

where we have made use of Eq. (5) with  $b = 2$  and  $F_{max}(x) = (1 - e^{-2x\gamma_{rd}^{-1}})^N$  is the CDF of  $\gamma_S^{max-min}$ .

This yields Eq. (15).

At high SNR, its first order expansion is given by

$$F_{max}(x) = (1 - e^{-2x\gamma_{rd}^{-1}})^N = (2x\gamma_{rd}^{-1})^N + o(x^{N+\epsilon}), 0 < \epsilon < 1$$

By simply algebraic manipulation of the above equation to obtain the PDF, and using Eq. (7), we can obtain its asymptotic BER as shown in Eq. (16).

### B. Proof of Theorem 2

We assume that all fading coefficients are independent. Let  $\gamma_i^{uj}(k) = \gamma_{rd}|h_{ri,u_j}(k)|^2$ . For each user  $j$ ,  $j = 1, 2$ , we rearrange  $\gamma_i^{uj}(k)$ ,  $i = 1, \dots, N$ , in an ascending order, denoted by  $\gamma_{(i)}^{uj}(k)$ , such that  $\gamma_{(1)}^{uj}(k) \leq \gamma_{(2)}^{uj}(k) \leq \dots \leq \gamma_{(N)}^{uj}(k) = \gamma_{S(j)}^{uj}(k)$ ,  $j = 1, 2$ . That is, we arrange  $\gamma_{(i)}^{uj}(k)$  as the  $(N-i+1)$ -th maximal in order. As we can see from Fig. 3, since  $\gamma_{S(j)}^{u1}(k)$  and  $\gamma_{S(j)}^{u2}(k)$  are independent, when  $\gamma_{S(1)}^{u1}(k)$  is the maximal among all  $\gamma_i^{u1}(k)$ ,  $\gamma_{S(1)}^{u2}(k)$  could be any  $q$ -th ( $q = 1, \dots, N$ ) maximal in order among all  $\gamma_i^{u2}(k)$ . That is, for any  $q$ ,  $q = 1, \dots, N$ , the probability that  $\gamma_{S(1)}^{u1}(k)$  is the  $q$ -th maximal in order is the same and equal to  $1/N$ . Similarly, when  $\gamma_{S(2)}^{u2}(k)$  is the maximum one among all  $\gamma_i^{u2}(k)$ ,  $\gamma_{S(2)}^{u1}(k)$  could be of any  $q$ -th maximal in order among all  $\gamma_i^{u1}(k)$ .

The average BER for D-RS-NC, denoted by  $P^{D-RS-NC}(\gamma_{rd})$ , can be calculated as

$$\begin{aligned} P^{D-RS-NC}(\gamma_{rd}) &= \frac{1}{2} E \left( \sum_{j=1}^2 Q \left( \sqrt{2\gamma_{uj}^{uj}(k)} \right) \right) \\ &= E \left( Q \left( \sqrt{\gamma_{S(1)}^{u1}(k) + \gamma_{S(2)}^{u1}(k)} \right) \right) \\ &= \frac{1}{N} \sum_{S(1)=S(2)} E \left( Q \left( \sqrt{\gamma_{S(1)}^{u1}(k) + \gamma_{S(2)}^{u1}(k)} \right) \right) \\ &+ \frac{1}{N} \sum_{S(1) \neq S(2)} E \left( Q \left( \sqrt{\gamma_{S(1)}^{u1}(k) + \gamma_{S(2)}^{u1}(k)} \right) \right) \\ &= \frac{1}{N} \sum_{S(1)=S(2)} E \left( Q \left( \sqrt{2\gamma_{(N)}^{u1}(k)} \right) \right) \\ &+ \frac{1}{N} \sum_{S(1) \neq S(2)} E \left( Q \left( \sqrt{\gamma_{(N)}^{u1}(k) + \gamma_{(q)}^{u1}(k)} \right) \right) \\ &= L_1 + L_2 \end{aligned} \quad (33)$$

The first term  $L_1$  can be calculated by following the similar procedure as for the S-RS-NC and is given by

$$\begin{aligned} L_1 &= \frac{1}{N} \sum_{S(1)=S(2)} E \left[ Q \left( \sqrt{2\gamma_{(N)}^{u1}(k)} \right) \right] \\ &= \frac{\sqrt{\gamma_{rd}}}{2N} \sum_{p=0}^N \binom{N}{p} (-1)^p \frac{1}{\sqrt{p + \gamma_{rd}}} \end{aligned} \quad (34)$$

Now let us calculate the second term  $L_2$ . We first find the moment generating function (MGF) of  $Z = X_1 + X_2 = \gamma_{(N)}^{u1} + \gamma_{(q)}^{u1}$ . Let us define  $w_l = \gamma_{(l)}^{u1} - \gamma_{(l-1)}^{u1}$  for  $l = 2, \dots, N$  and  $w_1 = \gamma_{(1)}^{u1}$ . It can be shown that  $w_l$  for  $l = 1, \dots, N$  are independent, and distributed according to [22]

$$f_{w_l}(w_l) = \frac{N-l-1}{\gamma_{rd}} \exp \left( -\frac{N-l-1}{\gamma_{rd}} w_l \right) \quad (35)$$

Then we have

$$\begin{aligned} Z &= \gamma_{(N)}^{u_1} + \gamma_{(q)}^{u_1} = \sum_{k=1}^N w_k + \sum_{k=1}^q w_k \\ &= 2 \sum_{k=1}^q w_k + \sum_{k=q+1}^N w_k \end{aligned} \quad (36)$$

The moment generate function (MGF) of  $Z$  is thus given by

$$\begin{aligned} M_Z(s) &= \int_0^\infty \dots \int_0^\infty \left( \prod_{k=1}^N f_{w_k}(w_k) \right) e^{sZ} dw_1 \dots dw_N \\ &= \left( \prod_{k=1}^q \int_0^\infty e^{2s w_k} f_{w_k}(w_k) dw_k \right) \\ &\quad \left( \prod_{k=q+1}^N \int_0^\infty e^{s w_k} f_{w_k}(w_k) dw_k \right) \\ &= \frac{N!(-1)^N}{2^q \gamma_{rd}^N} \prod_{k=1}^q \frac{1}{s - \frac{N-k+1}{2\gamma_{rd}}} \prod_{k=q+1}^N \frac{1}{s - \frac{N-k+1}{\gamma_{rd}}} \end{aligned} \quad (37)$$

By using the partial fraction expansion, Eq. (37) can be further expressed in Eq. (38).

Based on Eq. (38),  $L_2$  can then be calculated in Eq. (39), where  $\Psi_0 = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{\sin^4(\theta)}{(c_1 + \sin^2(\theta))(c_2 + \sin^2(\theta))} d\theta$  and it can be further calculated in (22).

By substituting (39) and (34) into (33), we can obtain the desired expression in Eq. (21).

To derive the high SNR approximation, we first evaluate the first order expansion of the PDF of  $Z = X_1 + X_2$  by applying a Taylor series expansion around the origin. This can be written as

$$f_Z(z) = \frac{f_Z^{(p)}(0)z^p}{p!} + o(z^{p+1}) \quad (40)$$

where  $f_Z^{(p)}(0)$  is the  $p$ -th derivative of  $f_Z(z)$  with respect to  $z$ , evaluated at zero, and  $p$  is the minimum integer value such that  $f_Z^{(p)}(0) \neq 0$ . Using the derivative property of the Laplace transform and the Initial Value Theorem, we can restate the equivalent problem as finding the minimum value of  $p$  such that

$$f_Z^{(p)}(0) = \lim_{s \rightarrow \infty} s^{p+1} M_Z(-s) \neq 0 \quad (41)$$

By substituting Eq. (37) into Eq. (41), we can show that  $p = N - 1$ , which gives

$$f_Z(z) = N 2^{-q} z^{N-1} + o(z^N) \quad (42)$$

Substituting Eq. (42) into Eq. (7), the asymptotic BER at high SNR can be written as

$$\begin{aligned} P^{D-RS-NC}(\gamma_{rd}) &= \frac{\Gamma(N + \frac{1}{2})}{2N\sqrt{\pi}} \gamma_{rd}^{-N} \\ &+ \sum_{q=1}^{N-1} \frac{2^{N-1} N \Gamma(N + \frac{1}{2}) 2^{-q}}{\sqrt{\pi} N^2} \gamma_{rd}^{-N} + o(\gamma_{rd}^{-N}) \\ &= \frac{(2^N - 1) \Gamma(N + \frac{1}{2})}{2N\sqrt{\pi}} \gamma_{rd}^{-N} + o(\gamma_{rd}^{-N}) \end{aligned} \quad (43)$$

This proves Theorem 2.

### C. Proof of Theorem 3

Let  $F_{\gamma_{sum,j}}(x)$  represent the CDF of  $\gamma_{sum,j}$ , then it is given by

$$F_{\gamma_{sum,j}}(x) = \frac{\gamma(N, Nx\gamma_{rd}^{-1})}{\Gamma(N)} = 1 - e^{-Nx\gamma_{rd}^{-1}} \sum_{p=0}^{N-1} \frac{(Nx\gamma_{rd}^{-1})^p}{p!}$$

The average BER of NC-No-RS can then be calculated as

$$\begin{aligned} P^{NC-No-RS}(\gamma_{rd}) &= \frac{1}{2} E \left( \sum_{j=1}^2 Q \left( \sqrt{2\gamma_{sum,j}(k)} \right) \right) \\ &= \frac{1}{2} \sum_{j=1}^2 \int_0^\infty Q(\sqrt{2x_j}) f_{\gamma_{sum,j}}(x_j) dx_j \\ &= \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-x}}{\sqrt{x}} F_{\gamma_{sum,j}}(x) dx \\ &= \frac{1}{2} - \frac{1}{2\sqrt{\pi}} \sum_{p=0}^{N-1} \frac{(N\gamma_{rd}^{-1})^p}{p!} \Gamma(p + \frac{1}{2}) (1 + N\gamma_{rd}^{-1})^{-(p+\frac{1}{2})} \end{aligned}$$

This proves Theorem 3.

### REFERENCES

- [1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity -Part I: system description," *IEEE Trans. Commun.*, vol. 51, pp. 1927-1938, Nov. 2003.
- [2] R. Ahlswede, N. Cai, S. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Inf. Theory*, vol. 46, no. 4, pp. 1204-1216, Jul. 2000.
- [3] J. N. Laneman, D. N. C. Tse and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, Dec. 2004, pp. 3062-2080.
- [4] A. Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 6, pp. 2020-2040, Jun. 2005.
- [5] C. Hausl, and J. Hagenauer, "Iterative network and channel decoding for the two-way relay channel," *Proc of ICC*, June 2006, pp. 1568 - 1573.
- [6] P. Popovski, and H. Yomo, "Physical network coding in two-way wireless relay channels," *Proc of ICC*, June 2007, pp. 707 - 712.
- [7] F. Xue, C.H. Liu and S. Sandhu, "MAC-layer and PHY-layer network coding for two-way relaying: achievable rate regions and opportunistic scheduling," *Proc of Allerton Conference 2007*, Sept 2007, pp. 396-402.
- [8] I. Baik and S. Y. Chung, "Network coding for two-way relay channels using lattices," *Proc of ICC*, May 2008, pp. 3898 - 3902.
- [9] R. Madan, et al., "Energy-efficient cooperative relaying over fading channels with simple relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, no. 8, pp. 3013 - 3025, Aug. 2008.
- [10] Y. Zhao, R. Adve, and T. J. Lim, "Improving amplify-and-forward relay networks: optimal power allocation versus selection," *IEEE Trans. Wireless Commun.*, vol. 6, No. 8, pp. 3114 - 3123, Aug. 2007.
- [11] R. Madan, et al., "Efficient cooperative relaying over fading channels with simple relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, No. 8, pp. 3013 - 3025, Aug. 2008.
- [12] Y. Li, B. Vucetic, Z. Chen and J. Yuan, "An improved relay selection scheme with hybrid relaying protocols," *Proc of GLOBECOM*, Nov. 2007, pp. 3704 - 3708.
- [13] H. A. David, *Order Statistics*, Jonh Wiley & Sons, Inc, 1970.
- [14] S. M. Alamouti, "A simple transmit diversity technique for wireless communications", *IEEE JSAC*, vol.16, no.8, pp.1451-1458, Oct 1998.
- [15] X. Bao and J. Li, "A unified channel-network coding treatment for user cooperation in wireless Ad-Hoc networks," *Proc of ISIT 2006*, July 2006, pp. 202 - 206.
- [16] M. Yu, J. Li, and R. S. Blum, "User cooperation through network coding," *Proc of ICC 2007*, June 2007, pp. 4064 - 4069.
- [17] P. Larsson, N. Johansson, K. E. Sunell, "Coded bi-directional relaying," *Proc of VTC 2006 Spring*, vol. 2, 2006, pp. 851 - 855.

$$\begin{aligned}
M_Z(s) &= \frac{N!(-1)^N}{2^q \gamma_{rd}^N} \sum_{k=1}^q \frac{1}{\left(s - \frac{N-k+1}{2\gamma_{rd}}\right) \prod_{j=1, j \neq k}^q \left(\frac{N-k+1}{2\gamma_{rd}} - \frac{N-j+1}{2\gamma_{rd}}\right)} \sum_{p=q+1}^N \frac{1}{\left(s - \frac{N-p+1}{\gamma_{rd}}\right) \prod_{j=q+1, j \neq p}^N \left(\frac{N-p+1}{\gamma_{rd}} - \frac{N-j+1}{\gamma_{rd}}\right)} \\
&= \frac{N!(-1)^N}{2\gamma_{rd}^2} \sum_{k=1}^q \sum_{p=q+1}^N \frac{1}{\prod_{j=1, j \neq k}^q (j-k) \prod_{m=q+1, m \neq p}^N (m-p)} \frac{1}{\left(s - \frac{N-k+1}{2\gamma_{rd}}\right) \left(s - \frac{N-p+1}{\gamma_{rd}}\right)}
\end{aligned} \tag{38}$$

$$\begin{aligned}
L_2 &= \frac{1}{N} \sum_{S_{(1)} \neq S_{(2)}} \mathbb{E} \left[ Q \left( \sqrt{\gamma_{(N)}^{u_1} + \gamma_{(q)}^{u_1}} \right) \right] \\
&= \sum_{q=1}^{N-1} \frac{(N-1)!(-1)^N}{2\gamma_{rd}^2} \sum_{k=1}^q \sum_{p=q+1}^N \frac{1}{\prod_{j=1, j \neq k}^q (j-k) \prod_{m=q+1, m \neq p}^N (m-p)} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{1}{\left(\frac{1}{2\sin^2(\theta)} + \frac{N-k+1}{2\gamma_{rd}}\right) \left(\frac{1}{2\sin^2(\theta)} + \frac{N-p+1}{\gamma_{rd}}\right)} d\theta \\
&= \sum_{q=1}^{N-1} (N-1)!(-1)^N \sum_{k=1}^q \sum_{p=q+1}^N \frac{1}{\prod_{j=1, j \neq k}^q (j-k) \prod_{m=q+1, m \neq p}^N (m-p) (N-p+1)(N-k+1)} \Psi_0
\end{aligned} \tag{39}$$

- [18] S. W. Kim, "Concatenated network coding for large-scale multi-hop wireless networks," *Proc of WCNC 2007*, Mar 2007, pp. 985 - 989.
- [19] S. Zhang, Y. Zhu, S. Liew and K. Letaief, "Joint design of network coding and channel decoding for wireless networks," *Proc of WCNC 2007*, Mar. 2007, pp. 779 - 784.
- [20] C. Hausl and P. Dupraz, "Joint network-channel coding for the multiple-access relay channel," *Proc of SECON*. 2006, vol. 3, Sept. 2006, pp. 817 - 822.
- [21] T. J. Oechtering, and H. Boche, "Bidirectional regenerative half-duplex relaying using relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 1879 - 1888, May 2008.
- [22] M. K. Simon and M. Alouini, *Digital communication over fading channels*, 2nd edition, *Jonh Wiley & Sons*, Inc, 2005.
- [23] Z. Wang and G.B. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *IEEE Trans. Commun.*, vol.51, no.8, pp.1389-1398, Aug. 2003.
- [24] J. Yuan, et al., "Distributed space-time trellis codes for a cooperative system," *IEEE Trans Wireless Commun.*, vol. 8, no. 10, pp. 4897-4905, Oct 2009.
- [25] C. Wang, et al. "Superposition-coded concurrent decode-and-forward relaying," *Proc of ISIT 2008*, July 2008, pp. 2390 - 2394.
- [26] S. X. Ng, Y. Li and L. Hanzo, "Distributed turbo trellis coded modulation for cooperative communications," *Proc of ICC 2009*, June 2009.
- [27] F. A. Onat, Y. Fan, H. Yanikomeroglu and H. V. Poor, "Threshold based relay selection in cooperative wireless networks," *Proc of IEEE Globecom*, Nov. 2008.
- [28] M. Eslamifar, C. Yuen, W. H. Chin, Y. L. Guan, "Max-Min Antenna Selection for Bi-Directional Multi-Antenna Relaying", *Proc of VTC-Spring 2010*, available at <http://www1.i2r.a-star.edu.sg/~cyuen/publications.html>.
- [29] M. Eslamifar, W. H. Chin, C. Yuen, and Y. L. Guan, "Performance Analysis of Two-Way Multiple-Antenna Relaying with Network Coding", *Proc of VTC-Fall 2009*, 2009.
- [30] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE JSAC*, vol. 24, no. 3, pp. 659-672, Mar. 2006.